LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034
B.Sc. DEGREE EXAMINATION – MATHEMATICS
FIFTH SEMESTER – NOVEMBER 2009 – 10
MT 5508 – LINEAR ALGEBRA

Max. Marks: 100
Duration: 3 hrs.

Section – A

Answer ALL the questions: **(10 x 2 = 20)**

1. Define a vector space *V* over a field *F*.
2. Show that the vectors (1, 1) and (-3, 2) in R^2 are linearly independent over *R*, the field of real numbers.
3. Define homomorphism of a vector space into itself.
4. Define an inner product space.
5. Normalise $(1 + 2i, 2 – i, 1 – i)$ in C^3 relative to the standard inner product.
6. Define eigen value and eigen vector.
7. Show that $A = \begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ is unitary.
8. If *A* and *B* are Hermitian, show that $AB + BA$ is Hermitian and $AB – BA$ is skew-hermitian.
9. State the Schwarz inequality.
10. Prove the parallelogram law in a inner product space *V*.

Section – B

Answer any FIVE questions: ` **(5 X 8 = 40)**

11. Let *V* be a vector space of dimension *n*, and let $v_1, v_2, v_3, \dots, v_r$ be linearly independent vectors in *V*. Prove that there exists $n – r$ new vectors $v_{r+1}, \dots, v_n$ in V such that $\{v_1, \dots, v_n\}$ is a basis of *V*.
12. Show that the set $\{1, x, x^2, \dots, x^n\}$ is a basis of the vector space $F[x]$ of all polynomials of degree atmost *n*.
13. Let *V* and *W* be two *n*-dimensional vector spaces over *F*. Then prove that any isomorphism *T* of *V* onto *W* maps a basis of *V* onto a basis of *W*.
14. If *V* is a finite-dimensional inner product space and if *W* is a subspace of *V*, then show that $V = W \oplus W^{\perp}$, the direct sum of *W* and its orthogonal complement.
15. Prove that $T \in A(V)$ is invertible if and only if *T* maps *V* onto *V*.

16. For $A, B \in F_n$ and $\lambda \in F$, prove that

(i) tr $(\lambda A) = \lambda$ tr A, (ii) tr $(A + B)$ = tr A + tr B, (iii) tr (AB) = tr (BA).

17. Prove that the eigen values of a unitary transformation are all of absolute value 1.

18. Prove that for any m x n matrix A over a field F, the row rank and column rank are equal.

Section – C

Answer any TWO questions: **(2 X 20 = 40)**

19. (a)The vector space V over F is a direct sum of two of its subspaces W_1 and W_2 if and only if $V = W_1 + W_2$ and $W_1 \cap W_2 = (0)$.Prove the statement.

(b)Prove that if V is a vector space of finite dimension and W is a subspace of V, then $\dim V/W = \dim V - \dim W$.

20. (a)Let V be a vector space of dimension n over F. Then prove that the dual space V^* also has dimension n.

(b)Prove that every finite-dimensional inner product space has an orthonormal set as a basis.

21. (a)Let V be a vector space of dimension n over F, and let $T \in A(V)$. If $m_1(T)$ and $m_2(T)$ are the matrices of T relative to two bases $\{v_1, \ldots, v_n\}$ and $\{w_1, \ldots, w_n\}$of V, respectively, then show that there is an invertible matrix C in F_n such that $m_2(T) = Cm_1(T)C^{-1}$.

(b)Let $V = R^3$ and suppose that $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ is matrix of $T \in A(V)$ relative to the standard basis $v_1 = (1, 0, 0)$, $v_2 = (0, 1, 0)$, $v_3 = (0, 0, 1)$. Find the matrix of T relative to the basis $w_1 = (1, 1, 0)$, $w_2 = (1, 2, 0)$, $w_3 = (1, 2, 1)$.

22. If $T \in A(V)$ has all its eigen values in F, then prove that there is a basis of V in which the matrix of T is triangular.